Diploma Programme
Programme du diplôme
Programa del Diploma

Mathematical studies
Standard level
Paper 1

Tuesday 10 May 2016 (afternoon)

1 hour 30 minutes

Candidate session number

Instructions to candidates

- Write your session number in the boxes above.
- Do not open this examination paper until instructed to do so.
- A graphic display calculator is required for this paper.
- A clean copy of the **mathematical studies SL formula booklet** is required for this paper.
- Answer all questions.
- Write your answers in the boxes provided.
- Unless otherwise stated in the question, all numerical answers should be given exactly or correct to three significant figures.
- The maximum mark for this examination paper is **[90 marks]**.

20EP01

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Please **do not** write on this page.

Answers written on this page will not
be marked.

Maximum marks will be given for correct answers. Where an answer is incorrect, some marks may be given for a correct method, provided this is shown by written working. Write your answers in the answer boxes provided. Solutions found from a graphic display calculator should be supported by suitable working, for example, if graphs are used to find a solution, you should sketch these as part of your answer.

1. The probability that Nikita wins a tennis match depends on the surface of the tennis court on which she is playing. The probability that she plays on a grass court is 0.4.
   The probability that Nikita wins on a grass court is 0.35. The probability that Nikita wins when the court is not grass is 0.25.

   (a) Complete the following tree diagram. [3]
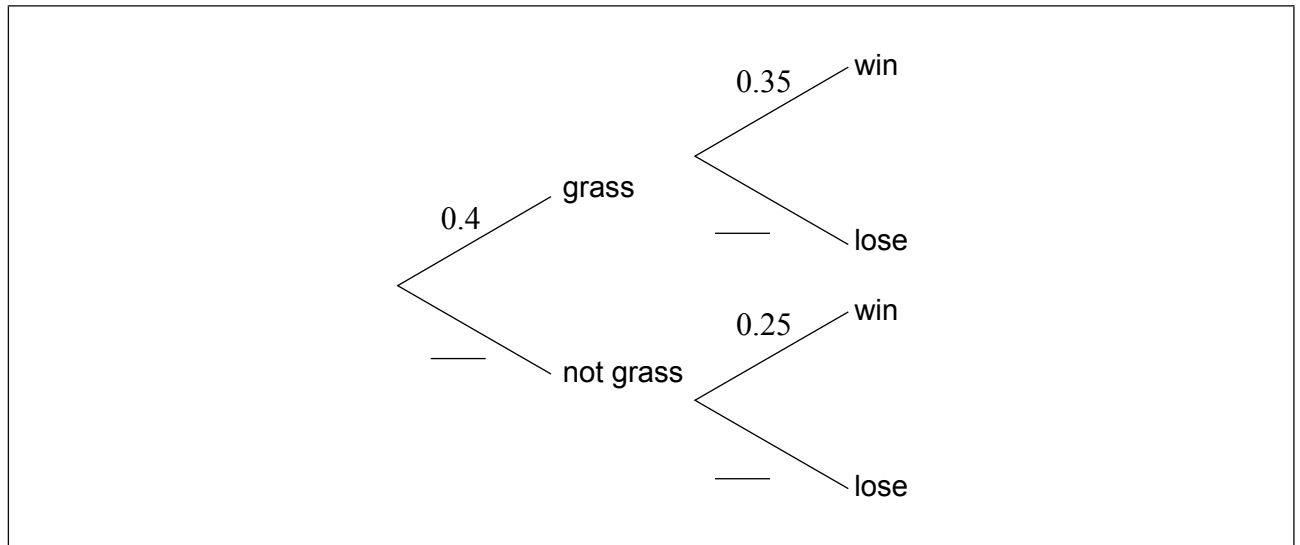
   (b) Find the probability that Nikita wins a match. [3]

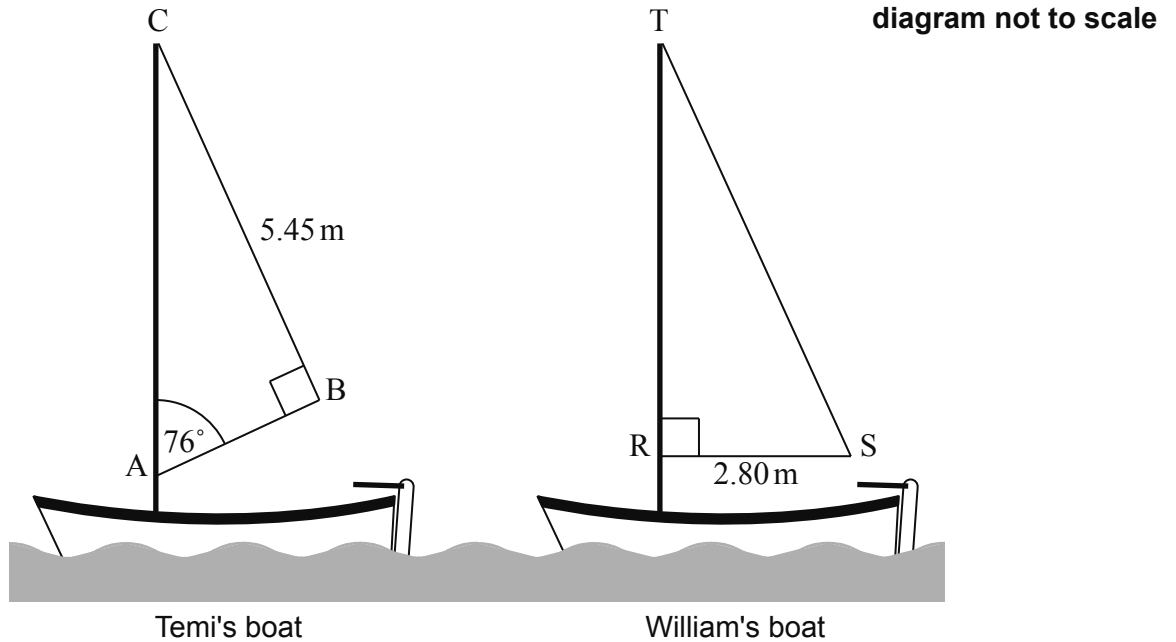   **Working:**

   Answer:

   (b) ....

Turn over

2.   Temi's sailing boat has a sail in the shape of a right-angled triangle, ABC. BC = 5.45 m,
angle CAB = 76° and angle ABC = 90°.

(a)   Calculate AC, the height of Temi's sail.   [2]

diagram not to scale

Temi's boat   William's boat

William also has a sailing boat with a sail in the shape of a right-angled triangle, TRS.
RS = 2.80 m. The area of William's sail is 10.7 m².

(b)   Calculate RT, the height of William's sail.   [2]

(c)   Calculate the size of angle RST.   [2]

(This question continues on the following page)

(Question 2 continued)

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

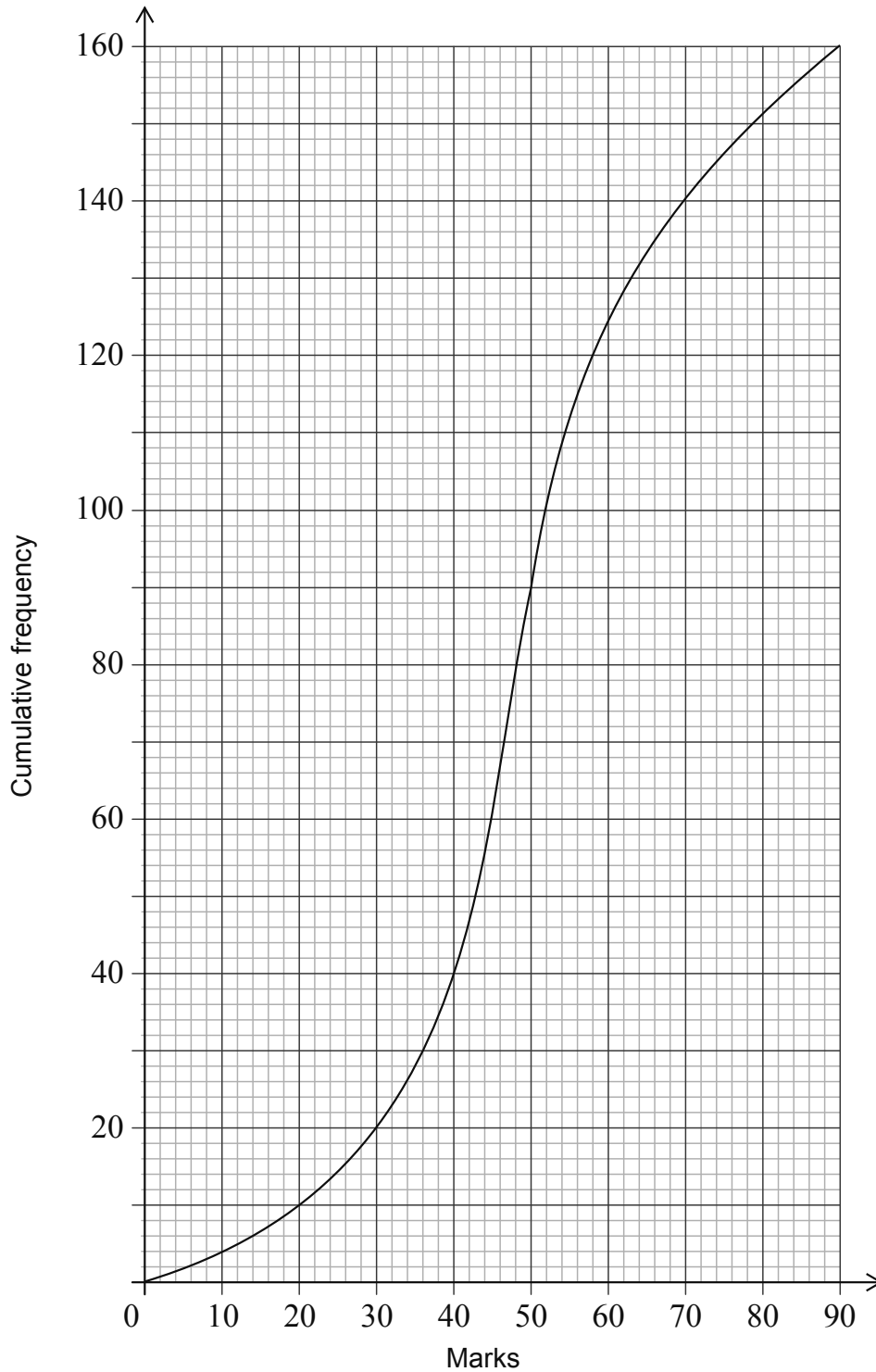**3.** In a school 160 students sat a mathematics examination. Their scores, given as marks out of 90, are summarized on the cumulative frequency diagram.

(a) Write down the median score. [1]

The lower quartile of these scores is 40.

(b) Find the interquartile range. [2]

(This question continues on the following page)

(Question 3 continued)

The lowest score was 6 marks and the highest score was 90 marks.

(c)   Draw a box-and-whisker diagram on the grid below to represent the students'
examination scores.   [3]

0   10   20   30   40   50   60   70   80   90   100

Marks

Working:

Answers:

(a) . . . .

(b) . . . .

Turn over
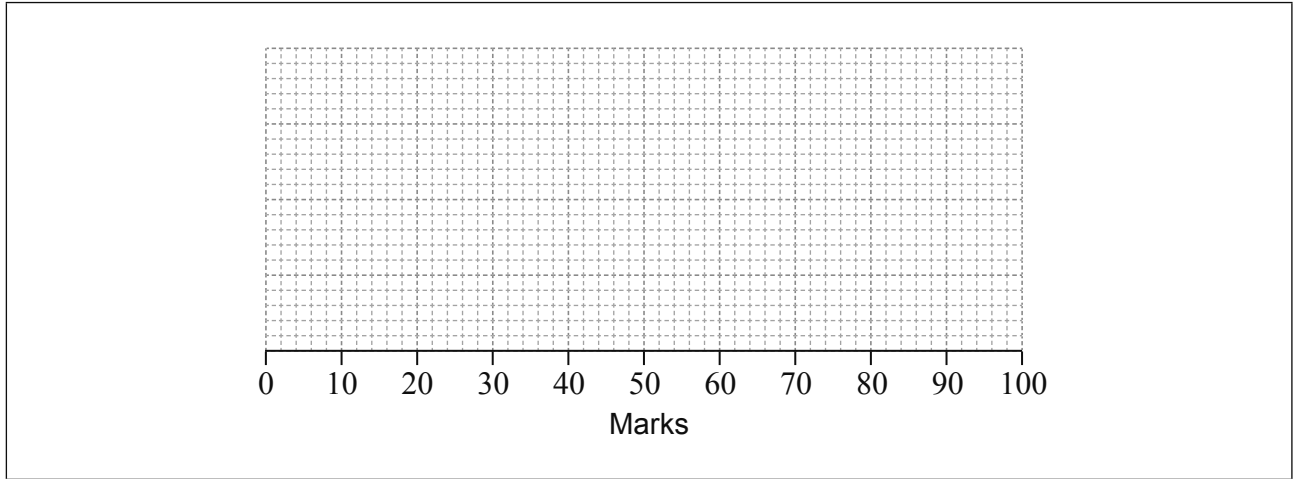
4.   FreshWave brand tuna is sold in cans that are in the shape of a cuboid with length $8\,\text{cm}$, width $5\,\text{cm}$ and height $3.5\,\text{cm}$. HappyFin brand tuna is sold in cans that are cylindrical with diameter $7\,\text{cm}$ and height $4\,\text{cm}$.

diagram not to scale
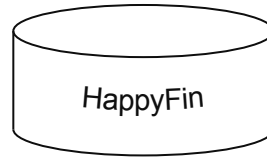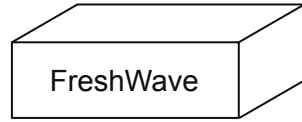
(a)   Find the volume, in cm^3, of a can of

   (i)   FreshWave tuna;

   (ii)   HappyFin tuna.   [4]

The price of tuna per cm^3 is the same for each brand. A can of FreshWave tuna costs 90 cents.

(b)   Calculate the price, in cents, of a can of HappyFin tuna.   [2]

Working:

Answers:

(a)   (i)   . . . .

   (ii)   . . . .

(b)   . . . .

5. Consider the following statements

$$z: x \text{ is an integer}$$
$$q: x \text{ is a rational number}$$
$$r: x \text{ is a real number.}$$

(a) (i) Write down, in words, $\neg q$.

(ii) Write down a value for x such that the statement $\neg q$ is true. [2]

(b) Write the following argument in symbolic form:
"If x is a real number and x is not a rational number, then x is not an integer". [3]

Phoebe states that the argument in part (b) can be shown to be valid, without the need of a truth table.

(c) Justify Phoebe's statement. [1]

Working:

Answers:

(a) (i) . . . .

. . . .

(ii) . . . .

(b) . . . .

(c) . . . .

. . . .

. . . .

6. One of the locations in the 2016 Olympic Games is an amphitheatre. The number of seats in the first row of the amphitheatre, u_1, is 240. The number of seats in each subsequent row forms an arithmetic sequence. The number of seats in the sixth row, u_6, is 270.

(a) Calculate the value of the common difference, d. [2]

There are 20 rows in the amphitheatre.

(b) Find the **total** number of seats in the amphitheatre. [2]

Anisha visits the amphitheatre. She estimates that the amphitheatre has 6500 seats.

(c) Calculate the percentage error in Anisha's estimate. [2]

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

7.　The equation of line L_1 is $y = 2.5x + k$. Point $A(3, -2)$ lies on L_1.

　　(a)　Find the value of k.　　　　　　　　　　　　　　　　　　　　　　　　[2]

　　The line L_2 is perpendicular to L_1 and intersects L_1 at point A.

　　(b)　Write down the gradient of L_2.　　　　　　　　　　　　　　　　　　　[1]

　　(c)　Find the equation of L_2. Give your answer in the form $y = mx + c$.　　　[2]

　　(d)　Write your answer to part (c) in the form $ax + by + d = 0$ where a, b and $d \in \mathbb{Z}$.　　[1]

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

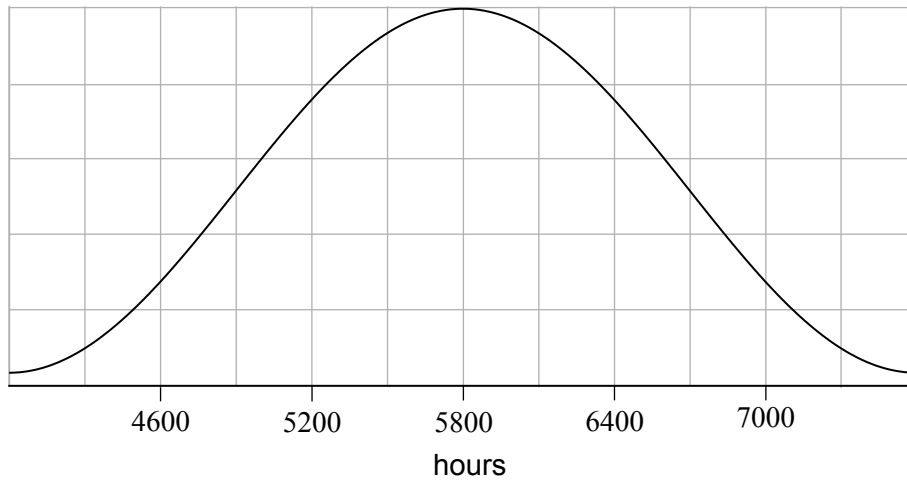(d) . . . .

8.   The lifetime, L, of light bulbs made by a company follows a normal distribution.
L is measured in hours. The normal distribution curve of L is shown below.

| | 4600 | 5200 | 5800 | 6400 | 7000 |

hours

(a)   Write down the mean lifetime of the light bulbs.   [1]

The standard deviation of the lifetime of the light bulbs is 850 hours.

(b)   Find the probability that $5000 \leq L \leq 6000$, for a randomly chosen light bulb.   [2]

The company states that 90% of the light bulbs have a lifetime of at least k hours.

(c)   Find the value of k. Give your answer correct to the nearest hundred.   [3]

Working:

Answers:

(a)   . . . .

(b)   . . . .

(c)   . . . .

9.   **In this question give all answers correct to the nearest whole number.**

Loic travelled from China to Brazil. At the airport he exchanged 3100 Chinese Yuan, CNY, to Brazilian Real, BRL, at an exchange rate of 1 CNY = 0.3871 BRL.
No commission was charged.

(a)   Calculate the amount of BRL he received.   [2]

When he returned to China, Loic changed his remaining BRL at a bank. The exchange rate at the bank was 1 CNY = 0.3756 BRL and a commission of 5% was charged. He received 285 CNY.

(b)   (i)   Calculate the amount of CNY Loic would have received if no commission was charged.

(ii)   Calculate the amount of BRL Loic exchanged when he returned to China.   [4]

Working:

Answers:

(a)   . . . .

(b)   (i)   . . . .

(ii)   . . . .

10. The manager of a travel agency surveyed 1200 travellers. She wanted to find out whether there was a relationship between a traveller's age and their preferred destination. The travellers were asked to complete the following survey.

Traveller survey

My age is:

25 or younger	26–40	41–60	61 or older

My preferred destination is:

New York	Tokyo	Melbourne	Dubai	Marrakech

A χ^2 test was carried out, at the 5% significance level, on the data collected.

(a) Write down the null hypothesis.   [1]

(b) Find the number of degrees of freedom.   [2]

The critical value of this χ^2 test is 21.026.

(c) Use this information to write down the values of the χ^2 statistic for which the null hypothesis is rejected.   [1]

From the travellers taking part in the survey, 285 were 61 years or older and 420 preferred Tokyo.

(d) Calculate the expected number of travellers who preferred Tokyo and were 61 years or older.   [2]

(This question continues on the following page)

(Question 10 continued)

Working:

Answers:

(a) . . . .

. . . .

(b) . . . .

(c) . . . .

(d) . . . .

11. Consider the function $f(x) = ax^2 + c$.

    (a)   Find $f'(x)$.   [1]

Point $A(-2, 5)$ lies on the graph of $y = f(x)$. The gradient of the tangent to this graph at A is -6.

    (b)   Find the value of a.   [3]

    (c)   Find the value of c.   [2]

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

12.   **In this question give all answers correct to two decimal places.**

Diogo deposited 8000 Argentine pesos, ARS, in a bank account which pays a nominal annual interest rate of 15%, **compounded monthly**.

(a)   Find how much **interest** Diogo has earned after 2 years.   [3]

Carmen also deposited ARS in a bank account. Her account pays a nominal annual interest rate of 17%, **compounded yearly**. After three years, the total amount in Carmen's account is 10 000 ARS.

(b)   Find the amount that Carmen deposited in the bank account.   [3]

Working:

Answers:

(a)   . . . .

(b)   . . . .

13. The golden ratio, r, was considered by the Ancient Greeks to be the perfect ratio between the lengths of two adjacent sides of a rectangle. The exact value of r is $\dfrac{1+\sqrt{5}}{2}$.

    (a) Write down the value of r

        (i)   correct to 5 significant figures;

        (ii)   correct to 2 decimal places.   [2]

Phidias is designing rectangular windows with adjacent sides of length x metres and y metres. The area of each window is $1\,\text{m}^2$.

    (b) Write down an equation to describe this information.   [1]

Phidias designs the windows so that the ratio between the longer side, y, and the shorter side, x, is the golden ratio, r.

    (c) Write down an equation in y, x and r to describe this information.   [1]

    (d) Find the value of x.   [2]

Working:

Answers:

(a) (i) . . . .

     (ii) . . . .

(b) . . . .

(c) . . . .

(d) . . . .

14. A population of 200 rabbits was introduced to an island. One week later the number of rabbits was 210. The number of rabbits, N, can be modelled by the function

$$N(t) = 200 \times b^t, \, t \geq 0,$$

where t is the time, in weeks, since the rabbits were introduced to the island.

(a) Find the value of b. [2]

(b) Calculate the number of rabbits on the island after 10 weeks. [2]

An ecologist estimates that the island has enough food to support a maximum population of 1000 rabbits.

(c) Calculate the number of weeks it takes for the rabbit population to reach this maximum. [2]

Working:

Answers:

(a) . . . .

(b) . . . .

(c) . . . .

15. A company sells fruit juices in cylindrical cans, each of which has a volume of $340\,\text{cm}^3$.
The surface area of a can is A cm^2 and is given by the formula

$$A = 2\pi r^2 + \frac{680}{r}\,,$$

where r is the radius of the can, in cm.

To reduce the cost of a can, its surface area must be minimized.

(a) Find $\dfrac{\mathrm{d}A}{\mathrm{d}r}$.   [3]

(b) Calculate the value of r that minimizes the surface area of a can.   [3]

Working:

Answers:

(a) . . . .

(b) . . . .